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Calibration Modeling of Nonmonolithic Wind-Tunnel Force Balances

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Experimental designs and regression models for calibrating nonmonolithic (multiple piece) internal wind-tunnel force balances were investigated through a case study that demonstrated fundamental deficiencies with a typical test schedule. It was found that the current calibration point selection method, which swept the design space two factors at a time, introduced a degree of correlation among model terms, depending on the model form. While using the statistical design of experiment performance metrics to analyze the deficiencies in the experimental design, it was also found that there were problems inherent to the model form itself that were independent of the design. An analysis of the calibration model, endorsed by the AIAA recommended practices document for nonmonolithic balances, lead to correlated response model terms due to overparameterization. Four new modeling strategies are proposed to overcome these challenges for nonmonolithic force balances.

Nomenclature

a_i	=	calibration model intercept
$b1_{i,j}$	=	coefficient estimate of main effects
$b2_{i,j}$	=	coefficient estimate of absolute value effects
b_{1a}	=	coefficient estimate of absolute value of main effect in analytical solution
b_{11}	=	coefficient estimate of pure quadratic in analytical solution
$c1_{i,j}$	=	coefficient estimate of pure quadratics
$c2_{i,j}$	=	coefficient estimate of partitioned absolute quadratic
$c3_{i,j,k}$	=	coefficient estimate of two-factor interactions
$c4_{i,j,k}$	=	coefficient estimate of absolute value of two-factor interactions
$c5_{i,j,k}$	=	coefficient estimate of partitioned absolute two-factor interactions
$c6_{i,j,k}$	=	coefficient estimate of partitioned absolute two-factor interactions
$\boldsymbol{D}$	=	calibration design matrix
$d1_{i,j}$	=	coefficient estimate of pure cubics
$d2_{i,j}$	=	coefficient estimate of absolute value of pure cubics
$d3_{i,m,j,k}$	=	coefficient estimate of three-factor interactions
F_j	=	force or moment, lb or in · lb
$\mathcal{R}$	=	correlation matrix
R_i	=	calibration model response (μV/V)
$\boldsymbol{X}$	=	model matrix
$\hat{Y}$	=	predicted response of an arbitrary model
z_i	=	generic factor setting in analytical solution
δ	=	indicator variable
Ψ	=	estimate of indicator variable coefficient

I. Introduction

A TYPICAL wind-tunnel internal balance is a six-degree-of-freedom force and moment transducer, capable of measuring an aerodynamic normal force, side force, axial force, pitching moment, yawing moment, and rolling moment by monitoring structural deformation with strain gages. The balance is said to be internal, because the instrument is mounted within the wind-tunnel model.

Figure 1 shows a typical six-degree-of-freedom internal wind-tunnel force balance. The metric (left) end of the balance connects to the aircraft model, while the nonmetric (right) end is attached to support structure in the wind tunnel, commonly known as the sting. There are three measuring sections that include two cage sections and an axial section. The cage sections are used to measure all forces and moments except axial force. The axial section measures axial force only. Typically, these sections are machined out of a solid block of material (called a monolithic design) to reduce mechanical hysteresis.

In this research, we focus on another type of balance that is fabricated by machining individual components and fastening them together mechanically; hence, the balance is a nonmonolithic structure. The mechanical joints exhibit a different response behavior than a monolithic balance. In particular, monolithic balances display a continuous response output when the applied load transitions from positive to negative or from negative to positive in direction. In contrast, the output of a nonmonolithic balance often spikes at the zero load point and commonly has a different slope on either side of the zero load point, as shown in Fig. 2.

This response behavior may not be limited to a change in slope; it could also affect a change in quadratic curvature, depending on the direction of the load. In addition, the change in response across the zero load point may also be different for each subdomain of the entire six-dimensional design space. When this situation exists, it means that the responses are dependent upon the polarity of all forces and moments. The AIAA recommended practices document (AIAA R-091-2003) states, "… it is not uncommon for the load/output relationship of balances, especially those of multipiece design, to exhibit some dependency on the sign of the strain in the measuring element. This asymmetry results in the need to determine and use different calibration coefficients according to the sign of the force or moment acting on the bridge, in order to achieve the best accuracy

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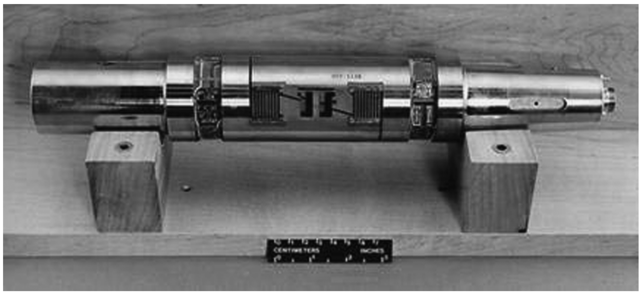

Fig. 1 Typical six-degree-of-freedom internal wind-tunnel force balance.

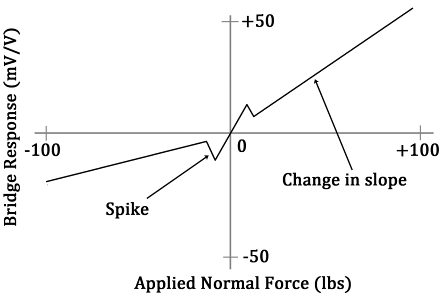

Fig. 2 Nonmonolithic force balance response due to applied force.

from the balance" [1], The asymmetry is a challenging phenomenon to capture, using traditional balance calibration modeling methods and its behavior is not fully understood. In this paper, we show that the recommended model suggested in R-091-2003, which seeks to address these issues, has significant statistical deficiencies. To improve the accuracy of nonmonolithic balances, alternative model approaches are proposed.

II. Current Practice

One of the goals of the recommended practices document (AIAA R-091-2003) is to standardize the calibration process, leading to a universal calibration matrix form that can be used for comparison between calibrations performed at different institutions [1]. The recommended practice for calibrating nonmonolithic balance suggests building a single regression model for each of the six degrees of freedom, as opposed to building multiple models in individual subdomains for each degree of freedom. The former is preferred over the latter, since it provides a simple, compact method for exchanging and implementing calibration matrices in various wind-tunnel facilities. The recommended practice suggests using an extension of the traditional response surface model with the inclusion of absolute value terms in order to model asymmetric behavior. The 97-term model is shown next in Eq. (1):

$$
\begin{aligned}
R_i = a_i &+ \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} b2_{i,j}|F_j| + \sum_{j=1}^{n} c1_{i,j}F_j^2 \\
&+ \sum_{j=1}^{n} c2_{i,j}F_j|F_j| + \sum_{j=1}^{n}\sum_{k=j+1}^{n} c3_{i,j,k}F_jF_k \\
&+ \sum_{j=1}^{n}\sum_{k=j+1}^{n} c4_{i,j,k}|F_jF_k| + \sum_{j=1}^{n}\sum_{k=j+1}^{n} c5_{i,j,k}F_j|F_k| \\
&+ \sum_{j=1}^{n}\sum_{k=j+1}^{n} c6_{i,j,k}|F_j|F_k + \sum_{j=1}^{n} d1_{i,j}F_j^3 + \sum_{j=1}^{n} d2_{i,j}|F_j^3| \qquad (1)
\end{aligned}
$$

In this model, terms that include an absolute value will act as a modifier to the similar effects that lack the absolute value. These model terms attempt to adjust the response surface depending upon the polarity of the applied forces and moments. However, the 97-term model suffers from poor statistical properties if all of the terms are included simultaneously: for example, the collinearity between model terms and the prediction variance of future observations, both of which suffer from overparameterization. The recommended practice suggests discarding terms in the model when necessary; however, there are certain terms in this model that should not coexist. To illustrate, a simple comparison is made between a quadratic response and the absolute value of a cubic response, shown in Fig. 3. By visual inspection of Fig. 3, it is obvious that the quadratic function and absolute value of a cubic function have similar characteristics.

Another similar example involves the quadratic term and the absolute value term. In Fig. 4, it appears that the degree of collinearity between terms is less than that of Fig. 3. The deviation of the quadratic curve from the absolute value response may be great enough to lead one to believe that these terms are capturing significantly different behaviors, thus justifying the inclusion of both in the overall model. To determine the degree of their ability to independently model unique behavior, the severity of their collinearity is assessed with statistical methods. In the following section, we review the particular statistical methods that are used to assess the experimental design and model form.

III. Model Quality Metrics

We refer to the collection of individual load combinations (design points) used to calibrate a force balance as the experimental design. The distribution of the design points determines the statistical quality metrics of the design. A model matrix is derived from the experimental design and the postulated regression model. The disciplines of the Design of Experiments and the Response Surface Methodology (RSM) provide a suite of statistical performance metrics for analyzing experimental designs. To illustrate, consider a calibration design matrix, a model matrix, and a regression model, shown in Eqs. (2a–2c), respectively:

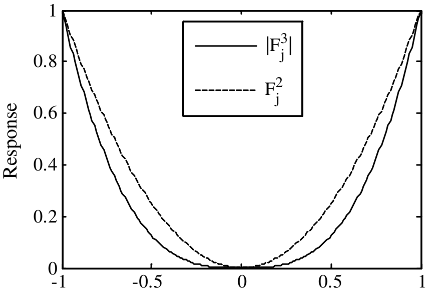

Fig. 3 Comparison between absolute value of cubic and quadratic terms.

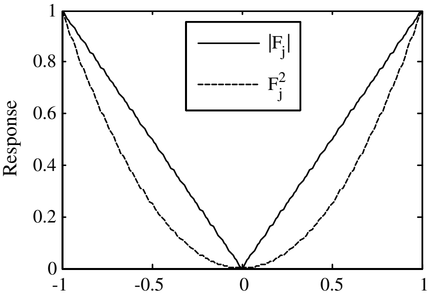

Fig. 4 Comparison between quadratic and absolute value terms.

$$\boldsymbol{D} = \begin{array}{cc} F_1 & F_2 \\ -10 & -20 \\ 10 & -20 \\ -10 & 20 \\ 10 & 20 \\ 0 & 0 \end{array} \tag{2a}$$

$$\boldsymbol{X} = \begin{array}{cccc} & F_1 & F_2 & F_1 \cdot F_2 \\ 1 & -10 & -20 & 200 \\ 1 & 10 & -20 & -200 \\ 1 & -10 & 20 & -200 \\ 1 & 10 & 20 & 200 \\ 1 & 0 & 0 & 0 \end{array} \tag{2b}$$

$$R = a + \beta_1 F_1 + \beta_2 F_2 + \beta_{12} F_1 F_2 \tag{2c}$$

Each row of the calibration design, $\boldsymbol{D}$, represents a different combination of loads in the calibration experiment. The number of columns in the model matrix is equal to the number of coefficients in the regression model. The predicted bridge output R is bridge output or response as a function of applied forces, F_1 and F_2.

A. Pearson Correlation Coefficient

For a given model matrix, the Pearson correlation coefficient r_{xy} quantifies the degree of the collinearity between columns x and y in the model matrix [2]:

$$r_{xy} = \frac{\Sigma xy - [(\Sigma x \Sigma y)/N]}{\sqrt{(\sum x^2 - [(\sum x)^2/N]\{\Sigma y^2 - [(\Sigma y)^2/N]\}}} \tag{3}$$

where $|r_{xy}| \leq 1$. Generally, values of r_{xy} near one indicate a strong positive linear association between x and y, whereas values of r_{xy} near -1 indicate a strong negative linear association. Values of r_{xy} near zero indicate little or no linear association between x and y. Computing the Pearson correlation coefficient between each column in a model matrix leads to a symmetrical correlation matrix $\mathcal{R}$, shown next:

$$\mathcal{R} = \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ & 1 & r_{23} & r_{24} & r_{25} & r_{26} \\ & & 1 & r_{34} & r_{35} & r_{36} \\ & & & 1 & r_{45} & r_{46} \\ & \text{sym} & & & 1 & r_{56} \\ & & & & & 1 \end{bmatrix} \tag{4}$$

B. Variance Inflation Factors

Variance inflation factors (VIF) are commonly used in place of correlation coefficients because of their more compact description of multicollinearity in a design [3]. The VIF measures the increased variance of a model coefficient due to the lack of orthogonality in the design. Orthogonality guarantees that each regression coefficient estimate is independent of any other. For each term in the model (excluding the intercept), we calculate a VIF, where a VIF of one indicates orthogonality. As a VIF increases, the collinearity of that term with others in the model increases. The VIFs of a design are computed as the diagonal component of the inverse of the Pearson correlation matrix. A general rule of thumb is that a VIF greater than 10 is an indication that multicollinearity may be unduly influencing least-squares estimates, while others suggest that this rule may be too lenient and recommend that a VIF should not exceed five. A limitation to the use of VIFs is that they cannot distinguish between several simultaneous multicollinearities, since they are only pairwise. When high VIFs are encountered, it is useful to refer back to the Pearson correlation matrix to determine which factors are correlated [3].

Using these statistical metrics, we investigate the collinearity of the 97-term model in the next section. An analytical solution is provided to illustrate why F_j^2 and $|F_j|$ cannot be orthogonal to each other and, therefore, cannot be estimated independently.

IV. Analytical Solution

For the regression model shown in Eq. (5), it can be shown that the absolute value term and quadratic term are unavoidably collinear by regressing the absolute value column x on the quadratic column y. The columns of x and y are shown in the model matrix $\boldsymbol{X}$ in Eq. (6):

$$R = b_{1a}|z| + b_{11}z^2 \tag{5}$$

$$\boldsymbol{X} = \begin{bmatrix} |z_1| & z_1^2 \\ |z_2| & z_2^2 \\ |z_3| & z_3^2 \\ \vdots & \vdots \\ |z_n| & z_n^2 \end{bmatrix} \quad (\text{columns } x, y) \tag{6}$$

Using ordinary least-squares regression (LSR), to regress the x column on the y column, the solution to the least-squares normal equations are given by the estimators [4]

$$b_1 = \frac{S_{xy}}{S_{xx}} \tag{7}$$

$$b_0 = \bar{y} - b_1\bar{x} \tag{8}$$

for the regression model

$$y = b_0 + b_1 x \tag{9}$$

where

$$S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})y_i \tag{10}$$

$$S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (7),

$$b_1 = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})y_i}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2} \tag{12}$$

To have the x column orthogonal to the y column, the estimator b_1 (slope) must be equal to zero. When b_1 equals one, this implies perfect collinearity between the x_i column and y_i column. The following relationship between the absolute value function and the quadratic function can be made:

$$|z_i|^2 = z_i^2 \tag{13}$$

or, in terms of x_i and y_i,

$$x_i^2 = y_i \tag{14}$$

Substituting Eq. (14) into Eq. (12), we obtain

$$b_1 = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})x_i^2}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2} \tag{15}$$

To show that collinearity is inevitable, the following relationships pertaining to the numerator and denominator of Eq. (15) are made:

$$\sum_{i=1}^{n}(x_i - \bar{x})x_i^2 \neq 0 \tag{16}$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 \neq 0 \tag{17}$$

To prove Eq. (16), we first develop the domain of z. A convenient coded variable (centered and scaled) is often used in regression modeling to define the levels of factor settings. The high-level setting is denoted as one, while the low-level setting is denoted as -1. Using coded variables, z is restricted by the following domain:

$$-1 \leq z \leq 1 \tag{18}$$

Since x is the absolute value of z, the domain of x is then

$$0 \leq x \leq 1 \tag{19}$$

And since y is z^2, we have

$$0 \leq y \leq 1 \tag{20}$$

At a minimum, for linear modeling, there must be at least two unique observations. Therefore, we can conclude

$$\bar{x} > 0 \tag{21}$$

Hence, the sum of the square of the difference between each observation, x_i and $\bar{x}$, must be positive, thus demonstrating that Eq. (17) cannot equal zero.

In the numerator of Eq. (15), there will be both positive and negative values in the summation expression $x_i - \bar{x}$. This suggests that the summation of the positive and negative terms could potentially equal zero, thus resulting in an orthogonal absolute value term and quadratic term. However, by decomposing the numerator of Eq. (15) into two parts [Eq. (22)], it can be shown that the numerator will always be positive:

$$\sum_{i=1}^{n}(x_i - \bar{x})x_i^2 = \sum_{i=j}^{k}(x_i - \bar{x})x_i^2 + \sum_{i=k}^{l}(x_i - \bar{x})x_i^2 \tag{22}$$

Where

$$\sum_{i=j}^{k}(x_i - \bar{x})x_i^2 \quad \text{for} \quad x_i < \bar{x} \tag{23}$$

and

$$\sum_{i=k}^{l}(x_i - \bar{x})x_i^2 \quad \text{for} \quad x_i > \bar{x} \tag{24}$$

For a design over the interval stated ($0 \leq x_i \leq 1$), the magnitude of $(x_i - \bar{x})$ in Eqs. (23) and (24) will be the same since the average can be seen as the center of mass of a group of design points in the x space and the two quantities as contributions from individual groups.

Next, it can be shown that x_i^2 in the domain of $0 \leq x_i \leq \bar{x}$ yields a significantly smaller value than in the domain $\bar{x} \leq x_i \leq 1$. Thus,

$$x_i^2(\text{for} \quad x_i > \bar{x}) > x_i^2(\text{for} \quad x_i < \bar{x}) \tag{25}$$

Summarizing the relationship between Eqs. (23) and (24),

$$\left|\sum_{i=k}^{l}(x_i - \bar{x})x_i^2\right| > \left|\sum_{i=j}^{k}(x_i - \bar{x})x_i^2\right| \tag{26}$$

Thus,

$$\Sigma_{i=j}^{k}(x_i - \bar{x})x_i^2 + \Sigma_{i=k}^{l}(x_i - \bar{x})x_i^2 > 0 \tag{27}$$

or

$$\sum_{i=1}^{n}(x_i - \bar{x})x_i^2 > 0 \tag{28}$$

Since the numerator of b_1 is always greater than zero, and the denominator is greater than zerom we can then make the following conclusion:

$$b_1 = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})y_i}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2} > 0 \tag{29}$$

Therefore, for a model including quadratic and absolute value terms, collinearity will always exist. The conclusion states that the column of quadratics will not be orthogonal to the column of absolute value terms; however, it does not give a degree of collinearity between the two. To illustrate the severity of collinearity, a numerical example is provided.

Given the model matrix,

$$\boldsymbol{X} = \begin{array}{c} \begin{matrix} \text{int} & x & |x| & x^2 \end{matrix} \\ \begin{bmatrix} 12 & x_1 & |x_1| & x_1^2 \\ 1 & x_2 & |x_2| & x_2^2 \\ 1 & x_3 & |x_3| & x_3^2 \end{bmatrix} \end{array} \tag{30}$$

To investigate the collinearity between the $|x|$ column and the x^2 column, the assumption is made that the design spans the range $0 \leq x \leq 1$, since it is desired to have design points at the extreme and at the origin for a favorable distribution of variance. This leads to the following model matrix:

$$\boldsymbol{X} = \begin{array}{c} \begin{matrix} \text{int} & x & |x| & x^2 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & x_2 & |x_2| & x_2^2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{31}$$

The Pearson's correlation coefficient is computed for columns $|x|$ and x^2 while varying x_2 from $0 \rightarrow 1$. Note that factor setting x_2 only varies from zero to positive one instead of minus one to positive one, because the sign will eventually turn positive when evaluated by the absolute value or quadratic function, yielding redundant results.

From Fig. 5, we see that the correlation between columns $|x|$ and x^2 is prevalent regardless of the level of factor setting x_2. A Pearson correlation coefficient above 0.96 corresponds to VIFs approaching infinity for both parameters. Next, the numbers of variable design points are incremented, while maintaining two constant design points at the extremes, as shown next:

$$\boldsymbol{X} = \begin{array}{c} \begin{matrix} \text{int} & x & |x| & x^2 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & x_2 & |x_2| & x_2^2 \\ 1 & x_3 & |x_3| & x_3^2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & |x_n| & x_n^2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{32}$$

The minimum correlation coefficient is found using a direct search computer algorithm that sweeps combinations of design configurations. The algorithm is run each time the design size is incremented. Figure 6 shows the minimum correlation coefficient for 20 different sized designs.

It is shown in Fig. 6 that the minimum correlation coefficient decreases as the design size increases; however, a tolerable degree of collinearity is never achieved. Even as the coefficient approaches 0.9, the VIFs for $|x|$ and x^2 still both approach infinity.

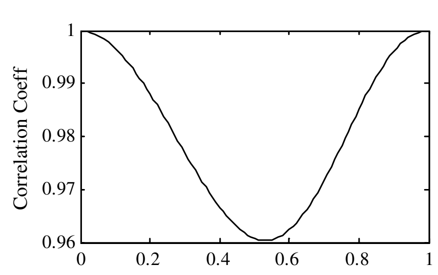

Fig. 5 Pearson correlation coefficient for the variable design of Eq. (31).

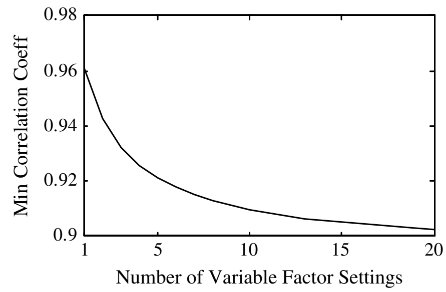

Fig. 6 Minimum Pearson correlation coefficient between columns $|x|$ and x^2 for different-sized designs.

V. Case Study of a Nonmonolithic Balance Calibration Design

Applying these concepts, we consider an experimental design for a nonmonolithic balance calibration and the recommended 97-term model. This sample load schedule varied two factors at a time. One factor is held constant while another is sequentially varied across its respective domain. This procedure is repeated until all possible two-factor combinations are executed. The load schedule consists of 1316 runs, resulting in a model matrix consisting of 1316 rows and 97 columns. From the model matrix, the following VIFs were computed, and a subset of these values are summarized in Table 1 for specific model terms.

By inspection of the Pearson correlation matrix, the following terms are most highly correlated (shown in Table 2).

The consequence of highly correlated model terms is increased prediction variance, which is undesirable for force balance calibration. The variance of a predicted value is a function of the model matrix and is calculated as follows [3,5]:

$$\mathrm{Var}\,[R(x)] = x^{(m)\prime}(X'X)^{-1}x^{(m)} \cdot \sigma^2 \tag{33}$$

where $x^{(m)}$ is a function of location and also a function of the model, X is the model matrix, and σ^2 is the mean squared error. For comparative purposes, we can assume that σ^2 equals one; therefore, in terms of standard error of prediction, Eq. (33) becomes [3,5]

Table 1 Mean VIF for individual model terms

Term	Approximate Mean VIF
$\Sigma_{j=1}^{n} b1_{i,j}F_j$	132.9
$\Sigma_{j=1}^{n} b2_{i,j}\|F_j\|$	148.0
$\Sigma_{j=1}^{n} c1_{i,j}F_j^2$	1,241.0
$\Sigma_{j=1}^{n} c2_{i,j}F_j\|F_j\|$	1,219.5
$\Sigma_{j=1}^{n}\Sigma_{k=j+1}^{n} c3_{i,j,k}F_jF_k$	1.3
$\Sigma_{j=1}^{n}\Sigma_{k=j+1}^{n} c4_{i,j,k}\|F_jF_k\|$	2.9
$\Sigma_{j=1}^{n}\Sigma_{k=j+1}^{n} c5_{i,j,k}F_j\|F_k$	1.9
$\Sigma_{j=1}^{n}\Sigma_{k=j+1}^{n} c6_{i,j,k}\|F_j\|F_k$	2.0
$\Sigma_{j=1}^{n} d1_{i,j}F_j^3$	1,084.2
$\Sigma_{j=1}^{n} d2_{i,j}\|F_j^3\|$	1,086.2

Table 2 Mean Pearson correlation coefficients for the most highly correlated terms

Term x	Term y	Approximate mean R_{xy}
$\Sigma_{j=1}^{n} b1_{i,j}F_j$	$\Sigma_{j=1}^{n} c2_{i,j}F_j\|F_j\|$	0.97
$\Sigma_{j=1}^{n} c1_{i,j}F_j^2$	$\Sigma_{j=1}^{n} b2_{i,j}\|F_j\|$	0.96
$\Sigma_{j=1}^{n} b1_{i,j}F_j$	$\Sigma_{j=1}^{n} d1_{i,j}F_j^3$	0.91
$\Sigma_{j=1}^{n} d2_{i,j}\|F_j^3\|$	$\Sigma_{j=1}^{n} b2_{i,j}\|F_j\|$	0.91
$\Sigma_{j=1}^{n} d2_{i,j}\|F_j^3\|$	$\Sigma_{j=1}^{n} c1_{i,j}F_j^2$	0.99
$\Sigma_{j=1}^{n} d1_{i,j}F_j^3$	$\Sigma_{j=1}^{n} c2_{i,j}F_j\|F_j\|$	0.99

$$\sqrt{\mathrm{Var}[R(x)]} = \sqrt{x^{(m)\prime}(X'X)^{-1}x^{(m)}} \tag{34}$$

The multicollinearity diagnostics presented in Tables 1 and 2 are derived from the model matrix, which depends on the experimental design and model formulation; therefore, the cause of the large VIFs could be attributed to either the experimental design or model form. Decoupling the amount of collinearity attributed by each is difficult, because no experimental design exists or could be found for the 97-term model that yields orthogonal coefficient estimates. While it may be possible to reduce the degree of collinearity for certain coefficients, based on our analytical derivation in the previous section, it cannot be made orthogonal by changing the design, since it is inherent to the model.

VI. Alternative Models

Four alternative methods for modeling the response of nonmonolithic force balances are presented here. The first approach suggests building individual models in separate subregions. The second is to use a pure cubic model to capture the entire response with a global, or single, model (as per the standard). The third is an absolute value model approach, and the fourth method uses an indicator variable scheme to fit a global regression model. Comparisons made between all approaches, including the 97-term model, used metrics that are calculated in the design planning and model formulation stage. Significant insight into model quality can be obtained in the planning phase of a calibration experiment. Careful consideration should be placed on coordinating the experimental design with the model formulation.

Both the independent model and indicator variable approach suggest partitioning the entire design space into subspaces. For a typical force balance, normal force is the highest load exerted on the joints of the multipiece balance and could be considered most likely to produce the most prevalent asymmetric behavior. For the independent models and indicator variable methods, the design space will be partitioned into subspaces dependent on the direction of the applied normal force. Therefore, the polarity of normal force defines a subregion regardless of the setting of the other five components.

A. Independent Models Approach

Using separate independent regression models and experimental designs for each subregion, designs exist that can be used to estimate the coefficients in Eq. (35) and provide VIFs approximately equal to one. A recommended design to implement in each subspace is a central composite design (CCD), which is commonly used to build the second-order RSM model, shown next [3]:

$$R_i = a_i + \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} c1_{i,j}F_j^2 + \sum_{j=1}^{n}\sum_{k=j+1}^{n} c3_{i,j,k}F_jF_k \tag{35}$$

By building separate models, the statistical metrics for each model can be calculated individually. If a CCD is chosen for each subspace, it can be shown that the quality metrics of each CCD model are acceptable. That is, all VIFs are one, and there is no collinearity present in the model [3]. One disadvantage of this approach is that it requires some form of a conditional statement in the wind-tunnel data reduction software so that the proper regression models are used, depending on the applied forces.

B. Pure Cubic Model Approach

The pure cubic model attempts to capture the asymmetric response behavior with a global model over the entire region of operability. Draper developed variance optimal rotatable designs for a pure cubic model [6,7]. These designs have three levels made up of a central composite and a Box–Benken design (BBD). The first orbit contains factorial points, while the second orbit has the axial points of the CCD and the design points of the BBD. A third and final orbit makes up the outermost axial points [8]. While Draper's design uses axial points necessary for a rotatable design, a spherical design may be more practical when extending Draper's design into six factors. It can be shown that Draper's design in six factors (228 runs) can support the pure cubic model shown in Eq. (36) with VIFs below three:

$$R_i = a_i + \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} c1_{i,j}F_j^2 + \sum_{j=1}^{n}\sum_{k=j+1}^{n} c3_{i,j,k}F_jF_k + + \sum_{j=1}^{n} d1_{i,j}F_j^3 \tag{36}$$

The pure cubic model of Eq. (36) was not developed for the specific application of classifying the asymmetric behavior of a nonmonolithic force balance; however, this model takes the approach of the 97-term model by including higher-order parameters to provide more flexibility in the response surface. Equation (36) allows for the use of standard regression model reduction methods without the ambiguity associated with the 97-term model. For these reasons, the pure cubic model could be a good candidate for modeling multipiece balance behavior.

From Table 3, it is clear that Draper's experimental design demonstrates superior orthogonality between model terms. Additionally, Draper's design benefits from a time and cost perspective by using only a fifth of the number of runs.

C. Absolute Value Model Approach

The third approach fits a global model using similar parameters to that of the 97-term model. This approach models main effects, the absolute value of the main effects, and two-factor interactions. The model, referred to as the absolute value model, is shown next:

$$R_i = a_i + \sum_{j=1}^{n} b1_{i,j}F_j + \sum_{j=1}^{n} b2_{i,j}|F_j| + \sum_{j=1}^{n} c2_{i,j}F_j|F_j| + \sum_{j=1}^{n}\sum_{k=j+1}^{n} c3_{i,j,k}F_jF_k \tag{37}$$

It is a reduced model compared with the 97-term model, having only 34 model terms. The strengths of this model lie in its ability to capture the discontinuity of first-order terms while maintaining the orthogonality. To illustrate the capabilities of the absolute value model, consider a purely linear single-factor response with a discontinuous slope that changes at the origin, shown in Fig. 7.

The quadratic term fits at the extremes; however, it is inadequate in capturing the behavior in the interior of the design region and is unable to account for the slope change. In balance calibration, first-order effects on the primary response, known as sensitivity coefficients, are usually dominant. Therefore, using an absolute value model may be the best approach to capture this dominant effect.

Once again, we should consider the model form and experimental design simultaneously. To demonstrate, consider the performance of the absolute value model using the 1316-run experimental design and Draper's 228-run experimental design, as shown in Table 4. Even though the absolute value model is a subset of the full 97-term model, improved designs should be considered.

D. Indicator Variable Approach

To model asymmetric behavior in separate subregions using a single model, a modeling method employing indicator variables [4] is considered. Indicator variables are used to model different

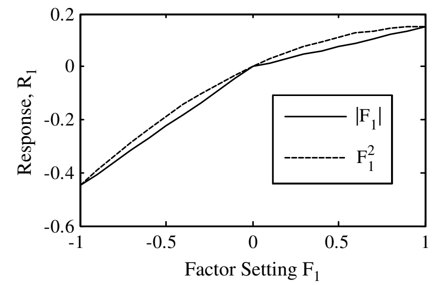

Fig. 7 Asymmetric slope modeling comparison.

processes that are independent of each other. Classically, the independent processes span the same domain where the indicator variable has no spatial dependence. In this approach, the indicator variable is used as a spatially dependent categorical variable that changes depending on the polarity of the applied normal force. Similar to the independent model approach, there is a design in each subregion. This approach is preferable over the independent models approach, since it can be presented in a single compact model that can be easily compared across institutions and more easily implemented in wind-tunnel data reduction. Consider a simple case to demonstrate the functionality of the indicator variable method. The single-factor model shown next contains an intercept β_0, a main effect β_1, and an adjusting term to the main effect Ψ_1:

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \Psi_1 \delta_1 X_1 \tag{38}$$

When using indicator variables, a guideline for choosing the proper indicator setup is that there should be one less indicator variable than there are groups or subspaces being modeled. In this case, there are two subregions; therefore, one indicator variable δ is used [4]. The indicator variable is dependent upon the applied normal force X_1 and is set up as follows:

$$\delta = -1, \qquad \text{if} \quad X_1 < 0 (\text{subregion 1}) \tag{39}$$

$$\delta = 1, \qquad \text{if} \quad X_1 > 0 (\text{subregion 2}) \tag{40}$$

The coefficient estimates, β_1 or Ψ_1, alone do not model the response of either region, but when Ψ_1 is subtracted from β_1, this corresponds to the response of subregion number one, and when Ψ_1 is added to β_1, this corresponds to the response of subregion number two. For example, consider the following single factor response:

$$\hat{Y} = 0.25X_1 - 0.15\delta_1 X_1 \tag{41}$$

Where, if $\delta = -1$ (subregion 1), the model becomes

$$\hat{Y} = 0.4X_1 \tag{42}$$

If $\delta = +1$ (subregion 2), the model becomes

$$\hat{Y} = 0.1X_1 \tag{43}$$

We can expand the previous example into higher dimensions while maintaining orthogonality. Since it is assumed only the slope of β_1 changes with the direction of the applied X_1, β_2 in this two-factor example remains constant across subregions. The former model is simply extended to incorporate a second factor, X_2, as follows:

Table 3 Comparison of mean VIF's for the pure cubic model

Term	1316-run schedule	228-run schedule
$\Sigma_{j=1}^n b1_{i,j}F_j$	7.5	2.0
$\Sigma_{j=1}^n c1_{i,j}F_j^2$	1.1	1.0
$\Sigma_{j=1}^n \Sigma_{k=j+1}^n c3_{i,j,k}F_jF_k$	1.3	1.0
$\Sigma_{j=1}^n d1_{i,j}F_j^3$	6.7	2.0

Table 4 Comparison of mean VIF's for the absolute value model

Term	1316-run schedule	228-run schedule
$\Sigma_{j=1}^n b1_{i,j}F_j$	16.7	4.0
$\Sigma_{j=1}^n b2_{i,j}\|F_j\|$	1.3	1.2
$\Sigma_{j=1}^n c2_{i,j}F_j\|F_j\|$	1.2	4.0
$\Sigma_{j=1}^n \Sigma_{k=j+1}^n c3_{i,j,k}F_jF_k$	15.9	1.0

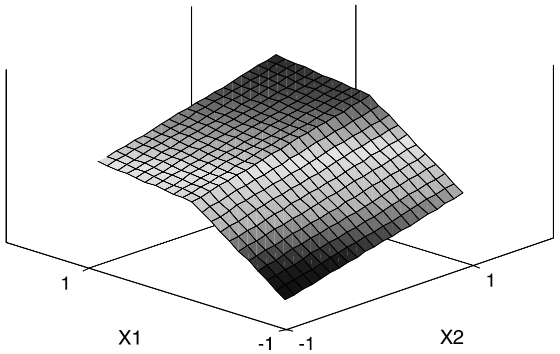

Fig. 8 Response surface with asymmetric slope in single direction.

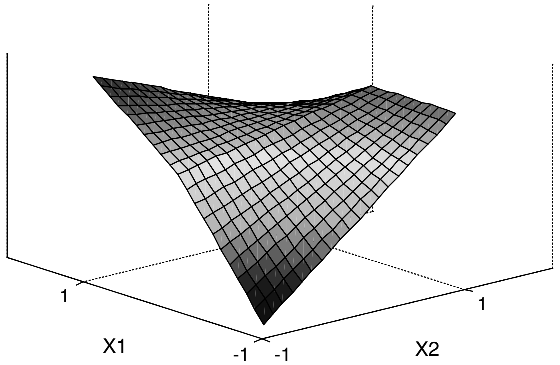

Fig. 9 Asymmetric slope in single direction with two-factor interaction effects.

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \Psi_1 \delta_1 X_1 \tag{44}$$

A representative response-surface modeling by the previous equation is shown in Fig. 8.

When choosing a design for this type of model, it seems appropriate to build a traditional factorial design in each subregion. It should also be noted that the δ column is omitted from the design matrix when performing LSR or calculating quality metrics, such as the correlation matrix or VIFs, since the intercept is constant across the entire domain. The resulting correlation matrix is a three by three identity matrix, while the VIFs are a row vector of ones, indicating orthogonality.

Using the same procedure, it is possible to extend the model to include six factors, with one linear adjusting term shown next in Eq. (45):

$$\hat{Y} = \beta_0 + \sum_{i=1}^{6} \beta_i X_i + \Psi_1 \delta_1 X_1 \tag{45}$$

The next example extends Eq. (45) further by introducing a two-factor interaction term, β_{12}, as shown next:

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \Psi_1 \delta_1 X_1 + \beta_{12} X_1 X_2 \tag{46}$$

Similar to the assumption made about the slope, β_2, it is also assumed that β_{12} is constant across the entire design space. A response surface representative of this model is shown in Fig. 9.

The same design is used for this model as was used for the previous example, with the addition of the two-factor interaction column, β_{12}. Once again, orthogonality is preserved, resulting in VIFs of one for all model terms.

Limitations to the indicator variable approach arise when trying to model higher-order terms, such as a quadratic, along with the adjusting term to the main effect. The adjustment term is analogous to the absolute value term in that they both effectively model the same phenomena and are correlated. Previously in this paper, it was shown analytically that quadratic and absolute value terms could not be orthogonal in a design. Similarly, it can be shown that a quadratic term and the adjustment to the main effect term cannot coexist orthogonally.

VII. Conclusions

Using well-known regression modeling multicollinearity diagnostics, it was shown that the AIAA-recommended 97-term calibration model for nonmonolithic balances contains correlated terms. Using analytical and numerical methods, it was shown that certain terms of the recommended model are unavoidably collinear, regardless of the experimental design employed. Therefore, we recommend that an addendum be made to the recommended practices document that clearly cautions users from estimating all 97 terms simultaneously.

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